Mechanical Engineering Formulas For Motion Control

Acceleration = $\dfrac{\text{Final Velocity} - \text{Initial Velocity}}{\text{Time}}$

Acceleration Torque = Moment of Inertia X Angular Acceleration

The torque exerted on an object is equal to the product of that object's moment of inertia times its angular acceleration. The angular acceleration is in the same direction as the torque. See Inertia and F=ma.

Circumference = Diameter x Pi (3.14) or Radius x 2Pi (6.28)

F=ma (Force = mass x acceleration)

Isaac Newton's second law of motion: the net force on an object is equal to the mass of the object multiplied by its acceleration.

FPM (feet per minute) = RPM x Circumference in feet

Gear ratio: reduces reflected inertia by the square of the reduction

<u>Example:</u> If reflected inertia to the motor is 225 lb. ft.^{2}, and a 15:1 reducer is added between the motor and the load, the reflected inertia is reduced by a factor of 225.

$15{:}1^2 = 15 \times 15 = 225$, Reflected Inertia $= \dfrac{225\ \text{lb. ft.}^2}{225} = 1\ \text{lb.ft.}^2$

Horsepower

A phrase coined by James Watt, the inventor of the steam engine. Horsepower is defined as work done over time. The exact definition of one horsepower is 33,000 lb.ft./minute. Put another way, if you were to lift 33,000 pounds one foot over a period of one minute, you would have expended one horsepower of energy.

HP=

$\dfrac{\text{Weight x Feet Per Minute}}{33{,}000}$

$\dfrac{\text{Torque(in.lbs.) x RPM}}{63{,}025}$

$\dfrac{\text{Torque(ft.lbs.) x RPM}}{5{,}252}$

1 HP = 746 Watts (continuous)

$\dfrac{\text{DC Motor Amps x Volts x \% Efficiency}}{746}$

$\dfrac{\text{Single Phase AC Motor Amps x Volts x Efficiency}}{746}$

$\dfrac{\text{3 Phase AC Motor Amps x Volts x Efficiency x Power Factor x 1.73}}{746}$

IPM (inches per minute) = PRM x Circumference in inches

Inertia and Acceleration Torque

Simplified, inertia refers to an object's amount of resistance to change in velocity. Isaac Newton stated in his first law of motion that unless a force is exerted upon it, a body in motion tends to remain in motion, and a body at rest tends to remain at rest. Rotational inertia, also referred to as "moment of inertia". It refers to the fact that a rotating rigid body maintains its state of uniform rotational motion. Its angular momentum is unchanged unless an external torque applied. This is also called conservation of angular momentum.

Inertia ($lb.ft.^2$)

WK^2 = W (weight in lbs.) x K^2 (radius of gyration)^2 = $lb.\ ft.^2$ (moment of inertia)

W = weight of the load in pounds

$$K^2 \text{ (radius of gyration)} = \frac{(\text{radius of disc. in feet})^2}{2}$$

As the radius of gyration increases, the moment of inertia of a the mass will increase by the square of the distance that the radius of gyration increases.

Example: If two rolls have equal weights, but Roll 1 diameter = 1 inch and Roll 2 diameter = 3 inches, it will take 9 times more torque to accelerate the 3 inch roll than the 1 inch roll.

Inertia ($lb.in.\ sec.^2$)

Inertia of a mass with a known weight (solid mass):

$$\mathbf{J} = \frac{1}{2} \times \frac{WR^2}{g}$$

Inertial of a mass with a known density (solid mass):

$$\mathbf{J} = \frac{1}{2} \times \frac{Pi \times L \times p \times R^4}{g}$$

J = inertia ($lb.in.sec.^2$)
W = weight (lbs.)
R = radius (inch)
L = length (inch)
p = density ($lb.in.^3$)
Pi = 3.14
g = gravitational constant (386 $in.sec.^2$)

Material	Density
Aluminum	.096 $lb.\ in.^3$
Steel	.280 $lb.\ in.^3$
Plastic	.040 $lb.\ in.^3$
Copper	.322 $lb.\ in.^3$
Wood	.029 $lb.\ in.^3$

To convert from $in.lb.sec.^2$ to $lb.ft.^2$, multiply $in.lb.sec.^2$ x 2.681

To convert from $lb.ft.^2$ to $in.lb.sec.^2$, multiply $lb.ft.^2$ x 0.3729

Acceleration Torque

These formulas will calculate the amount of torque required to accelerate or decelerate a load in a given period of time, or the amount of time it will take to accelerate or decelerate a load at a given torque.

To calculate torque when time is known=

$$\text{Inch lbs. torque} = \frac{.039 \times WK^2 \times (\text{Final RPM} - \text{Initial RPM})}{\text{Time in seconds to accelerate}}$$

$$\text{Lb. ft. torque} = \frac{WK^2\ (\text{Final RPM} - \text{Initial RPM})}{307.6 \times \text{Acceleration Time in Seconds}} =$$

To calculate the acceleration time when torque is known:

$$\text{Acceleration Time in Seconds} = \frac{.039 \times WK^2\ (\text{Final RPM} - \text{Initial RPM})}{\text{Torque in inch lbs.}} =$$

$$\text{Acceleration Time in Seconds} = \frac{WK^2\ (\text{RPM} - \text{Initial RPM})}{307.6 \times \text{Torque in ft. lbs.}}$$

Example: Calculate the WK^2 of a 36 inch diameter disk that weighs 200 lbs.

WK^2 = weight (lbs.) x (radius of gyration)^2

Diameter = 36 inches

Weight = 200 lbs.

$$\text{Radius of gyration}^2 = \frac{(\text{Radius of disc. in feet})^2}{2} = \frac{1.5\ \text{ft.}^2}{2} = \frac{2.25\ \text{ft.}}{2} = 1.125\ \text{ft.}$$

WK^2 = 200 lbs. x 1.125 ft. = 225 lb.ft.^{2}

Example: Calculate the torque needed to accelerate the disk.

The torque to accelerate the disc from 0-100 RPM in 5 seconds =

$$\frac{.039 \times WK^2 \times (\text{Final RPM} - \text{Initial RPM})}{\text{Time in seconds to accelerate}} =$$

$$\frac{.039 \times 225\ \text{ft.lbs.}^2\ (100\ \text{RPM} - 0\ \text{RPM})}{5\ \text{seconds}} = \frac{877.5}{5} = 175.5\ \text{inch lbs.}$$

To accelerate the disc from 0-100 RPM in 2.5 seconds =

$$\frac{.039 \times 225\ \text{ft.lbs.}^2\ (100\ \text{RPM} - 0\ \text{RPM})}{2.5\ \text{seconds}} = \frac{877.5}{2.5} = 351\ \text{inch lbs.}$$

To accelerate the disc from 0-200 RPM in 5 seconds =

$$\frac{.039 \times 225\ \text{ft.lbs.}^2\ (200\ \text{RPM} - 0\ \text{RPM})}{5\ \text{seconds}} = \frac{1755}{5} = 351\ \text{inch lbs.}$$

To accelerate the disc from 0-200 RPM in 2.5 seconds =

$$\frac{.039 \times 225\ \text{ft.lbs.}^2\ (200\ \text{RPM} - 0\ \text{RPM})}{2.5\ \text{seconds}} = \frac{1755}{2.5} = 702\ \text{inch lbs.}$$

Example: Calculate the time the disc is accelerated from 0-200 RPM with 500 inch lbs. of torque.

$$\frac{.039 \times WK^2\ (\text{RPMf} - \text{RPMi})}{\text{Torque in Inch Lbs.}} = \text{Acceleration Time in Seconds}$$

$$\frac{.039 \times 225\ \text{ft.lbs.}^2\ (200\ \text{RPM} - 0\ \text{RPM})}{500\ \text{inch lbs.}} = \frac{1755}{500} = 3.51\ \text{seconds}$$

Radius of Gyration:

A mathematical term which represents the distance from the axis to a point where, if the entire mass was concentrated there, its moment of inertia would be the same. Radius of gyration is used as the index for mass distribution for calculating moment of inertia.

The sweet spot of a **baseball bat** is it's radius of gyration. When we are discussing a rotating object we sometimes consider a location where the mass could be concentrated, without affecting the angular momentum; this is called the radius of gyration. A good example of this is a baseball bat. There is a spot on the bat, usually 4/5 of the distance from the handle to the end of the bat that is called the "sweet spot." When a ball is hit at that spot, there is no shock to the hands swinging the bat.

A batter has better control with a straight stick than a baseball bat because the radius of gyration is closer to the batter's hands. But, since the radius of gyration is closer to the batter's hands on a straight stick than on a baseball bat, the batter won't get the benefit of a higher moment of inertia, and therefore will have a harder time hitting the long ball. The disadvantage of a baseball bat is that since most of the weight is centered at the end of the bat, it requires more strength from the batter to accelerate and control the bat. As with a golf club, swing speed results in more energy going into hitting the ball, and therefore longer distances.

A batter who wishes to hit "see and eye" singles will probably want to use a shorter and lighter bat than a home run hitter, who will choose the longest and heaviest bat that the hitter can control.

Golfers refer to swing weight, but that is an incorrect term. They are actually referring to inertia, and the radius of gyration of the golf club.

Examples of where radius of gyration:

Sweet spot on a baseball bat
Sweet spot on a golf club
Car or bicycle wheel
Seesaw
Spinning figure skater
Diver rotating off the high dive
Gymnast on the high bar
Spinning marching band baton

RPM (revolutions per minute) = $\frac{\text{Lineal Velocity}}{\text{Circumference}}$

Torque

Torque is a twisting force. It causes a shaft to rotate. It is measured by the combination of load or pull multiplied by the distance from the centerline of a shaft or a pivot point. It is generally measured in the United States in foot lbs., inch lbs., or ounce inches. The metric unit of measure is Newton Meters.

Torque =

Force x Radius

$\text{Force}_{\text{(ounces)}} \times \text{Radius}_{\text{(inches)}}$ = oz. inches of Torque

$\text{Force}_{\text{(pounds)}} \times \text{Radius}_{\text{(inches)}}$ = inch lbs. of Torque

$\text{Force}_{\text{(pounds)}} \times \text{Radius}_{\text{(feet)}}$ = foot lbs. of Torque

Inch Lbs. Torque = $\frac{\text{Horsepower} \times 63{,}025}{\text{RPM}}$

Foot Lbs. Torque = $\frac{\text{Horsepower} \times 5{,}252}{\text{RPM}}$

Moment of Inertia X Angular Acceleration

The torque exerted on an object is equal to the product of that object's moment of inertia times its angular acceleration. The angular acceleration is in the same direction as the torque.

Velocity = $\frac{\text{Distance}}{\text{Time}}$